To the Commission on the reform of the electoral law and the modernisation of Parliament's work

Application of a voting system simulator for the analysis of ideas for the reduction of the Size of the Bundestag

– Preliminary results –

Thorkell Helgason, Kristjan Jonasson, und Lilja Steinunn Jonsdottir[1][2][3]

Abstract

A group of researchers at the University of Iceland has developed simulation software, an *election simulator*, that can be used to test different voting systems, existing or constructed, to test the quality of such systems, especially for parliamentary elections.

Since a reform of the Bundestag electoral law is imminent, the group has dealt with such electoral systems this summer, i.e. with systems consisting of constituencies together with Land lists. First, the work was focused on a single federal state. The Free State of Bavaria was selected for this purpose. The 2021 Bundestag election was used as the starting point of 10,000 simulated election results, with a coefficient of variation of 30% for the first votes and 10% for the second votes.

Some electoral systems have been tested, especially those that abolish the current overhang while respecting as much as possible the demand for full proportional representation between the parties based on the second votes. Four systems have been selected that have in common that there is no overhang, whereby the target number of Bundestag mandates of 598 is also respected. One of them is the *trench system*, with which proportional representation cannot be achieved. Furthermore, a system based on the principle of electoral systems in *Scandinavia* will be presented. It maintains the majority vote in the districts as well as the *trench system*, but treats the Land lists differently, thus reducing the disregard for proportional representation. Thirdly, the system set out in the Commission's *key points* has been tested. Full proportional representation is achieved, but with inevitable deviations from the majority vote. The fourth system examined is a variant of the cornerstone system, here called the *advantage system*, which mitigates the distortions in the constituencies.

As further work, it would be important to also choose data from other federal states or neutral examples of fictitious countries. It would also be necessary to treat all federal states together. The group already has ideas on how to implement this. It would also be interesting to include the idea in the *key points* about replacement votes in the election simulator. The group is ready for further work, including in cooperation with the Commission's experts.

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1 Prof. Emeritus University of Iceland, former Secretary of State and BPresident. — The next item is the joint debate on the following motions for resolutions:

2 Professor of Mathematics at the University of Iceland.

3 Master-Student at eth zurich.

I Introduction

For several years, a group of researchers at the University of Iceland has been working on the development of a so-called election system simulator.

The task of this software is to be able to check different electoral systems for parliamentary elections. Although the focus was originally on the electoral systems in the Scandinavian countries, it is also applicable to a system such as the one for the election of the Bundestag.

The simulator is not a forecasting tool for predicting the possible outcome of future elections, but a tool for measuring the quality of an electoral system; whereby several characteristics are calculated that make it possible to compare the systems with each other, knowing that no electoral system is perfect.[4]

As a basis for the evaluations, the simulator generates thousands – or as many as desired – randomized election results with a given mean and a selected coefficient of variation of a probability distribution (such as gamma, beta or equal distribution). As a starting point, an election result must be entered as an expected value around which the simulated election results are generated. This can be a historical election result, the average of some of them or a very fictitious result.

Why simulated election results and not just a time series of historical results? The answer is, on the one hand, that the latter are not numerous enough to achieve statistically significant quality features. On the other hand, however, it is not politically helpful to focus too much on previous results. Comparisons between such quality features of different electoral systems based on the same electoral results – historical or simulated – must be evaluated with reservation, because the voters and the parties adapt in some way to each electoral system.

The simulator offers a wide range of electoral systems – notonly all the usual distribution rules, such as those of D'Hondt and Sainte-Laguë, but also and in particular different methods for the distribution of balancing mandates with the purpose of avoiding overhang and at the same time achieving the most proportional distribution of seats possible. Reference and basis is the optimal method of Balinski-Demange, which can also be called [5]*entropy optimization method*. This method is based – in the case of countymandates – on selecting the candidates whose product of their votes is as large as possible; or the sum of the logarithms of the votes. It revolves around the method whose solution can be found with [6]*the algorithm of alternating scaling* of Pukelsheim and its employees. As is well known, this method is the only one that has certain self-evident quality characteristics: e.g. negative voting rights cannot arise or unwanted influence of "third parties" is excluded ("independence from irrelevant alternatives"). However, because this method is quite complicated to use, the simulator has various other, simpler

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4 The simulation modell von Behnke from 2009 can be compared with the one presented here (Behnke, Joachim: Überhangmandate bei der Bundestagswahl 2009. An estimate with simulations. In: Zeitschrift für Parlamentfragen, 2009 (40, 2): 620 – 636). At Behnke, the focus is on predicting the overhang in the upcoming Bundestag elections with the conclusion that the "reform of the electoral law is urgently needed".

5 Michel Louis Balinski / Gabrielle Demange: „An axiomatic approach to proportionality between matrices.“ Mathematics of Operations Research 14 (1989) 700-719.

Michel Louis Balinski / Gabrielle Demange: „Algorithms for proportional matrices in reals and integers.“ Mathematical Programming 45 (1989) 193-210.

6 Thorkell Helgason / Kurt Jörnsten: „Entropy of proportional matrix apportionments.“ Norwegian School of Economics and Business Administration, Institute of Finance and Management Science, Working Paper 4/94. Bergen-Sandviken, 1994.

methods, all of which can be seen as approximations of the above-mentioned optimal method, including the method mentioned in point 5b in the [7]*Commission's key points*.[8]

The simulator was used to examine the present – and a few more – ideas for reforming the election of the Bundestag. The following reports on the preliminary results.

The simulator does not yet allow the introduction of first preferences, as proposed in point 5a in the *key points* . However, these could be added relatively easily. The problem, however, is the data basis. In the following, therefore, this interesting nuance is not taken into account. In addition, the simulator is not yet adapted to states in general. That is why we refer to individual federal states. In this preliminary report, Bavaria is chosenas the "main country" of overhang mandates. Any other state can be simulated in the same way. In addition, it would be interesting to construct an "average" federal state as a basic model in order to get a certain distance from the individual cases.

It should be emphasized that this study focuses on mathematical-statistical aspects of the electoral systems. Therefore, among other things, no position is taken on the constitutionality of the presented electoral systems. The same applies to political consequences or reactions of voters. Both adapt in some way to any electoral system.

In the following, the results of the 2021 Bundestag election are often referred to. All data on this were taken from the relevant publication of the Federal Elector, to which reference is made in the following under "BwL". Names, abbreviations and order of the Länder, constituencies and parties are taken from this publication.[9][10]

II Overall distribution of mandates at federal level

As a preliminary stage of the distribution of mandates between district candidates and on Land lists, it must be available how the seat quotas of the Länder and the parties are found, above all in what order and also whether Länder or parties have priority or not.

There seem to be three possibilities for this overall distribution:

- **PL: Distribution of mandates by party and within them by country.** First, the mandates are distributed among the parties according to the nationwide second votes, provided that they exceed the 5% hurdle (with the exception of the South Schleswig Voters' Association, SSW). Then, within each party, the sub-distribution to the Länder follows, again after their second votes. This order is preferred in the *vertices*, see point 3. It has the disadvantage that the number of mandates of each country depends on the nationwide outcome of the elections and can vary from election to election – in our case from simulation to simulation.
- **LP: Distribution of mandates by country and within those by party.** Here the order is reversed: First, the mandates are distributed according to the population figures among the states,

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[7] See z.B. Friedrich Pukelsheim: „Proportional Representation“, Second Edition, Chapters 14-15, Springer, 2017.

[8] Interim report of the Commission on the reform of the electoral law and on the Modernisation of Parliament's work – Vertices. https://www.bundestag.de/resource/blob/903330/498c43d8485fc6bf2511dc54d232d77e/K-Drs-029-Eck-punkte-zum-Zwischenbericht-data.pdf

[9] The Federal Returning Officer: Election to the 20th German Bundestag on 26 September 2021, Issue 3, Final Results by Constituency. https://www.bundeswahlleiter.de/dam/jcr/cbceef6c-19ec-437b-a894-3611be8ae886/btw21_heft3.pdf

[10] The order of the parties corresponds to the FONESn Totalnumberin the Nationwide Second votes in the 2017 elections. From pure Typographic For reasons, we shorten the name DIE LINKE to „Linke" and GRÜNE on "Grüne".

which corresponds to the table in chapter 6.1.1 in the FONES for the 2021 Bundestag election. With this order, the parties are the "[11]Leidenden": Their total number of mandates does not always correspond exactly to the nationwide result of the election. Nevertheless, the present study practically assumes this sequence, because the simulations refer to individual countries.

- **BP: Proportional distribution of mandates by country and party.** Here, the distribution of mandates among the individual Land lists of the parties is regarded as a bidimensional problem. It is prescribed that both the countries get their mandates as in the distribution LP and the parties as in PL. Table 1 shows the output achieved with the *entropy optimization method*. Of course, other and simpler methods can be used. But then the balance between the countries on the one hand and the parties on the other would not be guaranteed. But in all cases, the sum conditions would be respected.

The outcome of all three distributions in the event of the 2021 Bundestag election can be found in Table 1. The distribution method BP is used there as a reference. Deviations of the two other distribution methods are shown with color code.

III Prerequisites, simplifications and definitions

In order to be able to use the election simulator with regard to the German electoral law reform, the following is assumed – partly in view of the political discussion in and around the Commission and the guidelines in the *key points*, but also for purely technical reasons:

- The total number of mandates will be determined and must not be dependent on the election result, as is the case now. This means that "overhang mandates" are to be abolished. The fixed number 598 is assumed.
- Before the election, the number of mandates of individual Länder is available (93 in Bavaria),as well as the division of these seats in the one-man districts (46 in Bavaria) and on the Land lists (47 in Bavaria).
- The relevant federal result 2021 shows which parties are above the nationwide 5% hurdle. These are CDU, SPD, AfD, FDP, Left, Greens and CSU. For the sake of simplicity, other parties or individual candidates are not considered.
- A two-vote system is adopted, i.e. each voter has a first and a second vote. In this study, the second votes in the country concerned are the basis for the overall distribution of all seats in the country to the parties. This targeted number of mandates of each of the parties is hereinafter referred *to as the seat contingent* of the party.[12][13]
- If the number of seat quota mandates is exceeded in a distribution method, we speak in this context – as is usually the case – of an *overhang*, *overhang mandates* and *overhang parties*.

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[11] If the compensatory mandates are added, the order will be changedso that the end result is in the style of order PL.

[12] The election simulator offers more Possibilities at: a) The sum of the first votes (as in state elections in Baden-Württemberg and Saarland). (b) Sum or average of the sum of the first votes and the second votes (Bavarian state elections).

[13] Here is also a deviation from the *Vertices*. In point 3, it is proposed that "the number of seats of the parties is first determined at the federal level (so-called upper distribution). [...] The number of seats of a party determined in this way is then distributed to the Party's Land lists in proportion to the second votes it has obtained in the Länder (so-called sub-distribution)." As already said, the voting simulator is still an not in States coupled in this sense adapted; hence this derogation, which is not for the purpose of of the study as a whole – quality testing of electoral systems – is contradictory.

- Whenever it comes to the distribution of seats or mandates, only the Sainte-Laguë/Schepers rule is used. There seems to be quite broad agreement on this in German politics. As already mentioned, any other of the common distribution rules could also be used.

Two terms in our application need to be clarified:

- We will be talking about direct mandates or seats if their distribution depends only and solely on the election result in the constituency in question; in no way on the results in other constituencies – in the case of the one-man circles not on the second votes.
- On the other hand, we call seats or mandates that are not independent in theabove sense *compensation mandates*. These are used to compensate for the disproportionality caused by the direct seats. The list mandates are in the generalcompensation mandates; not least if compensatory seats are added, as in the current electoral system.

IV Examined electoral systems for the elections to the Bundestag

In the present study on the reform of the electoral law, the following three basic systems have been examined, under the conditions set out in Chapter III.

A. Ditch system: As now, the district mandates are only awarded to the candidate with the most votes on the basis of the respective first votes. Regardless of this, the list mandates are distributed on the basis of the second votes. The total number of mandates is simply the sum of those from the counties and the state mandates for each party.

B. Scandinavian system: The second votes are not unconditionally the basis for the distribution of mandates to the parties. The district mandates are awarded, as in the *trench system*, on the basis of the first votes. The list mandates are based on the second votes, but after the allocation of the total number of district mandates. This can be described in such a way that the seat quotas resulting from the second votes are reduced in order to avoid overhang. This comes at the expense of perfect proportionality. This principle is used in the Scandinavian countries (including Iceland, but not finland, where all mandates are direct county mandates).[14][15]

C. Traffic light systems: The second votes are the basis for the overarching distribution of the overall mandates to the parties, i.e. the seat quotas. The district mandates are all considered as compensatory mandates, for the purpose that their total number for each party does not exceed the party's contingent ofseats.

In the *cornerstones* , emphasis is placed on this path; therefore, we allow ourselves to speak of the "traffic light system". But here there is more than one version and that's where the election

[14] In these Nordic countries (except Finland), the compensatory mandates are not on Land lists, but linked to the constituencies. In addition there is no Second votes; the sum of the Kreisstimmen plays their role, so as in Baden-Württemberg and Saarland.

[15] This variant, which are available here *Skandinavisch* called becomes, could also be used as a *MMP*electoral system („Mixed Member Proportional“) marked become. According to dem Manual of IDEA, „Electoral System Design: The New Internatioonl IDEA Handbalso" from 2005, coincides whose Definition very good with ourr; see this Neckline:

> **Mixed Member Proportional (MMP)** – A *mixed system* in which all the voters use the first *electoral system*, usually a *plurality/majority system*, to elect some of the representatives to an elected body. The remaining seats are then allocated to parties and groupings using the second *electoral system*, normally *List PR*, so as to compensate for disproportionality in their representation in the results from the first *electoral system*.

But because the definition from *MMP* in the Literature Very much different is, becomes the markingg *Scandinavian* beibehalten.

simulator comes into play. It offers about ten variants on how the compensation mandates can generally be distributed. In the case of one-man constituencies, it is reduced to four possibilities, three of which we present here as relevant:

a. **We** call the corner variant the version described in vertices 4 and 5b; however, now described somewhat differently: The district mandates are awarded according to the size of the percentage of votes from top to bottom, but within the upper limits determined by the second votes, i.e. a party is no longer eligible after its seat quota stock has been exhausted. [16][17]

b. **Relative lead**. In each circle, the ratio between the votes of the most popular candidate and the one with the second most votes is calculated. Due to this characteristic, the district mandates are distributed as in the *corner variant*, i.e. according to the amount of these ratios, within the upper limits given by the seat quotas.[18]

c. **Optimal solution**. The distribution of district mandates comes from the previously mentioned *entropy optimization.* Because all districts are now one-man constituencies, this solution can be described as meaning that the product of the votes of the elected candidates should be the highest. This optimal solution is hardly recommended because it is quite opaque. However, it is the only one that guarantees certain quality features. That's why it serves as a reference solution. Unfortunately, however, in the time available to us, it has not been possible to bring this variant to an end.

The goals of all three variants of the *traffic light systems* are clearly formulated in Behnke's text:[19]

1. "Proportional representation between the parties is strictly adhered to."
2. "The personnel selection element must be designed in such a way that it cannot come into conflict with the proportional character of the electoral system."
3. "The Bundestag's standard size of 598 seats is strictly adhered to."

In all three variants, the list mandates are simply differences, i.e. for each party the contingent of seats with deduction of the total number of district mandates won by it.

The *corner variant* is the simplest of the three. Together with all the "greedy" methods mentioned in optimization theory, it has the disadvantage that bottlenecks can occur and the last mandates have to be awarded due to quite small votes.

The method of *relative advantage* has been developed to avoid precisely such bottlenecks: the candidate who has the largest relative advantage over his successor is given priority. This is to avoid having to access the successor later in the process in this circle, because the party of the now best no longer has a mandate available. The method is, so to speak, an "accident avoidance method"! Computational experiments show that this method very often gives the same solution

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[16] The description can be found in the present *Vertices* not quite finished. That's why a little bit has to be interpreted here.

[17] In order to avoid overhang, this variant has been used since a change in the law in 2014 in the elections to the Swedish *Parliament* used. See https://www.lagboken.se/Lagboken/start/forvaltningsratt/vallag-2005837/d_2229861-sfs-2014_1384-lag-om-andring-i-vallagen-2005_837. The wording in the Swedish law is almost identical to that ofn the *Vertices* 4 and 5b.

[18] In the election simulator there are several editions of this method, which lead to the same result in the case of one-man constituencies.

[19] Alreadyachim Behnke - Wikipedia: "Statement on the enlargement of the Bundestag, on the meeting on 2 June of the Commission on the reform of the electoral law and the modernisation of the Parliamentary work", Commission document, K-Drs-017-Prof-Dr-Behnke-Statement-Reduction-Bundestag-II-data.

as the [20]*optimal* one. If these are only one-man constituencies and given that there are only two parties, then the Method of *Relative Advantage* gives the same distribution of mandates as the *Optimal Method*.

The *corner variant* and the method of the *relative lead* are described here in a "constructive" way, i.e. the district mandates are distributed in a certain order according to prescribed characteristics: in the first variant according to the amount of the vote shares, in the second according to the relative lead of the candidate with the most votes in relation to the one with the second most. The order of these two methods can be seen in Table 3 for the 2021 Bundestag election in Bavaria.

Both methods can also be described as "corrective" methods: first, the district mandates are unconditionally awarded to the candidate with the most votes (as is now the case) and then overhangs are abolished by awarding the overhang mandates to other candidates: seats are changed, first where the characteristic in question is smallest, etc. The "corrective", equivalent version of the *Relative Advantage* can be seen in Table 4.[21]

The *corner variant* and the relative *advantage* method are both free of "negative voting rights", but both suffer from possible "third party influence". It is to be expected that the relative *advantage* method will be less susceptible to this. However, this needs to be investigated in more detail.

All three systems, A, B and C, strictly adhere to the template that no overhang mandates are generated and therefore no compensatory mandates are necessary. This is not the case with conventional systems. Nevertheless, it is interesting to have such (simplified) systems as a comparison:

D. Overhang system: On the basis of the second votes, the overarching distribution of the total mandates to the parties is carried out, i.e. the seat quotas are determined. Regardless of this, the district mandates are only awarded on the basis of the first votes, as is the case now. The final result for each party is the maximum of the sum of the district mandates and the seat quota mandates. The list mandates are differences as in most other systems.

E. Compensation system: First as with the overhang system. If overhang mandates are created, i.e. more mandates are awarded than the originally planned 598, the number of total seats in the country (and then as list mandates) will be increased until the overhang mandates disappear.[22]

The dialing simulator is not yet suitable for systems D and E. These systems are therefore not dealt with further.

V Explanatory example

The systems introduced in Chapter IV can be illustrated and compared with a simple example. Seven mandates are to be awarded here, three constituency mandates and four list mandates. There are three parties.

The final distribution of mandates is summarised in the following overview:

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[20] The Idee follows the so-called *Vogels-Approximation* to solve the *Transport-Problems*. See Reinfeld, N.V., Vogel, W.R. 1958. Mathematical Programming, 59–70. Englewood Cliffs, New Jersey: Prentice-Hall.

[21] That the "constructive" and "corrective" versions of these two methods are equivalent applies to one-man constituencies, but not in general if each of the counties has multiple mandates. If it is too Erstpräferenzen , in many respects the corrective version is more logical; also with regard to the *Eckvariante*.

[22] That is, at the federal level, which is ignored in this description after this study focuses on individual federal states.

Erklärendes Beispiel

Drei Wahlkreise: Ost, Mitte, West
Drei Parteien: I, II, III
Vier Listenmandate

Eingangsdaten

Erststimmen

	I	II	III	Total
Kreisstimmen Ost	70	120	200	390
Mitte	50	125	200	375
West	200	190	50	440
Total	320	435	450	1205

Zweitstimmen

	I	II	III	Total
Landeslisten	550	445	220	1215

	I	II	III	Mandate
Sitzkontingente	3	3	1	7

Ampelsystem; Eckvariante

		I	II	III	Mandate
	Ost		1		1
	Mitte			1	1
	West	1			1
Direktmandate	Total	1	1	1	3
Listenmandate als Differenz		2	2		4
Gesamtmandate, gleich Sitzkontingenten		3	3	1	7

Ampelsystem; Relativer Vorsprung

		I	II	III	Mandate
	Ost			1	1
	Mitte		1		1
	West	1			1
Direktmandate	Total	1	1	1	3
Listenmandate als Differenz		2	2		4
Gesamtmandate, gleich Sitzkontingenten		3	3	1	7

Grabensystem

	I	II	III	Mandate
Ost			1	1
Mitte			1	1
West	1			1
Total	1		2	3
Listenmandate gem. Zweitstimmen	2	1	1	4
Gesamtmandate	3	1	3	7
Abweichung vom Sitzkontingent		-2	2	4

Ampelsystem; Optimale Lösung

		I	II	III	Mandate
	Ost			1	1
	Mitte		1		1
	West	1			1
Direktmandate	Total	1	1	1	3
Listenmandate als Differenz		2	2		4
Gesamtmandate		3	3	1	7

Skandinavisches System

	I	II	III	Mandate
Ost			1	1
Mitte			1	1
West	1			1
Total	1		2	3
Listenmandate als Differenz	2	2		3
Gesamtmandate	3	2	2	7
Abweichung vom Sitzkontingent		-1	1	2

Überhangsystem

	I	II	III	Mandate
Direktmandate	1		2	3
Listenmandate als Differenz	2	3		3
Gesamtmandate	3	3	2	8
Überhang			1	1

Ausgleichssystem

	I	II	III	Mandate
Direktmandate	1		2	3
Listenmandate als Differenz	3	3		3

The detailed calculations of the above abbreviated version are shown in Table 5, including explanations.[23]

VI Simulation Basics

As already mentioned before, the simulations refer to individual federal states. First and second votes are randomised for the country concerned. In each simulation, therefore, the total mandates of each state party are calculated as well as the distribution to the individual districts and their state list; all in accordance with the relevant electoral system, which is subject to examination.

Expected values of the simulated number of votes are the results of the 2021 Bundestag election. For simulations in the Free State of Bavaria, the votes are listed in Table 2. All number of votes is randomized with the gamma distribution. No correlation is introduced between the individual numbers, neither

[23] In fact, in this simple example, it has not been possible to find number of votes, in which the three Traffic light variants different district candidates show. Here is the *Relative lead* the same distribution as the *Best Method*. There are also numbers of votes that lead to equality between the *Eckvariante* and the *Optimaln Method* lead.

between the first votes and the second votes, nor between the first votes in the individual circles among themselves. This does not correspond to reality, but here it must be emphasized once again that the election simulator is not intended to predict election results, but to put systems through their paces. A fairly wide range of election results is necessary for this, but of course within a realistic framework. In this context, the determination of a coefficient of variation is important. In the following simulations, this is chosen with 0.3 for individual first votes, but smaller, i.e. 0.1 for the second votes.[24][25]

In the right half of Table 2, standard deviations give an overview of the variability these requirements give for the number of votes in the example of Bavaria.

10,000 simulated election results were generated. With such a large number of simulations, the calculated averages are very accurate. The error in all average numbers is, with 95% confidence, less than 0.5%.

VII Results of the simulations

So far, the study has covered the following electoral systems:

A. Trench system
B. Scandinavian system
C. Ampelsysteme
    a. Eckvariante
    b. Relative advantage

For purely technical reasons, the *optimal variant* can unfortunately not be shown at the moment.

The voting simulator yields a lot of results that can be downloaded both from the simulator's website and from Excel files. In the following sections, the most important of these results are highlighted, not least with illustrations.

VII-1 Potential overhang

In the current distribution of mandates in Bavaria based on the 2021 election results, the CSU causes 11 potential overhang mandates, i.e. district mandates in surplus to the party's seat quota or *district mandate seat quota* = 45-34 = 11. The starting point, as in the current electoral law, is that the district mandates are distributed as directly elected mandates, as is the case in the *trench system* and the *Scandinavian system*.

In the 10,000 simulated election results, the outcome is slightly different. Despite quite large variations in the number of votes, there is no overhang among the other parties, i.e. the overhang remains exclusively with the CSU. However, the quantity of the overhang is on average much smaller than in 2021. On average, there are "only" 6.5 overhang mandates. The outcome of the 2021 election was therefore an exception. This is illustrated in Figure 1.

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[24] If there is no "splitting" and the first votes between the Constituenciesn are independent, should the coefficient of variation of the second votes equal be the, what comes out when the number of Constituencies by the square root of the Number of counties in the country concerned dividiert becomes. For Bavaria,e therefore thebe Divisor be quite large, namely $\sqrt{46}$, d.h. almost 7. However, since the requirements are not whole are durable, only a ratio of 3 is used.

[25] Behnke (see footnote) 4) uses as standard deviation 1.5 to 2.0 percentage points in relation to the parties' share of second votes. If, for example, a party receives 20% of the second votes (which is not far from the average share of the parties above the 5% hurdle), then the 2.0 percentage points correspond to the used here Variationskoeffizient from 0,1.

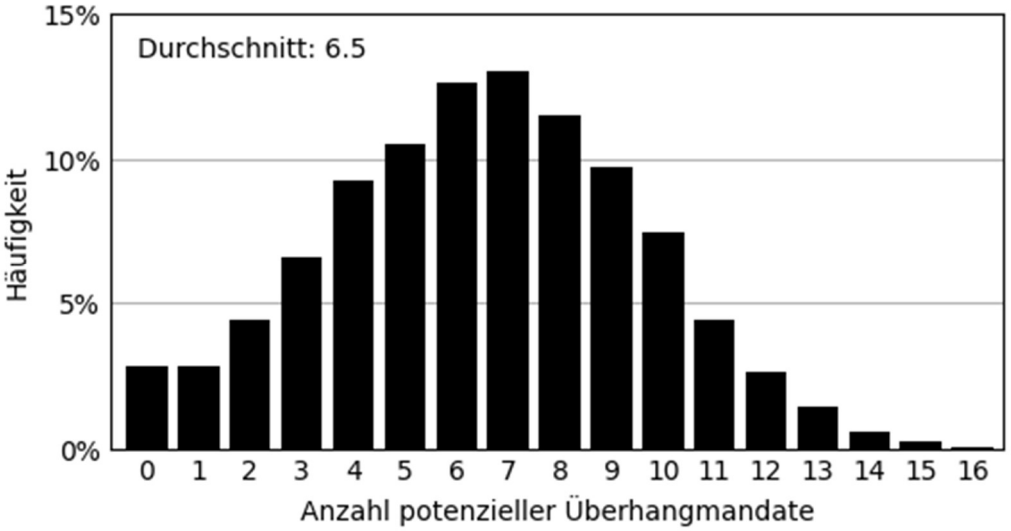

Figure 1: Frequency of potential overhang. Simulated results based on the Bundestag elections in Bavaria in 2021.Potential overhang arises if the majority election of direct mandates of one party (or several) results in more mandates than the party is entitled to due to the contingent of seats. In all these simulations, all overhang mandates in Bavaria fall to the CSU.

In the current electoral law, overhang leads to the enlargement of the Bundestag beyond the standard size of 598 mandates. It should be emphasized here once again that there is no overhang in all the systems covered here.

VII-2 Proportionality / Disproportionality

Apart from the abolition of the overhang, the declared goal of the traffic light coalition is to achieve full proportionality - also called *proportional representation* - i.e. that each party receives the number of mandates that its contingent of seats prescribes on the basis of the second votes. This is fully achieved in the [26]*traffic light systems* , but, as is to be expected, neither in the *trench system* nor in *Scandinavian*. The deviations of these two systems from full proportionality are shown in the following figures .

The disproportionality is caused by the overhang in the distribution of the district mandates to the respective majority candidates, as Figure 1 shows. This is common to both systems. The difference is how far the distribution of list mandates can overcome the overhang. Because the origin of the overhang in Bavaria can only be found in the CSU, only this party is overrepresented and then at the expense of the other parties.

In the *trench system*, the overhang is not compensated in any way. On the contrary, the disproportionality is rather reinforced by the fact that the list mandates are distributed independently on the basis of the second votes. Figures 2 and 3 show the result of the simulations in this context. Figure 2 shows in a histogram the frequency of surplus mandates at the CSU in the *trench system*. The average is just under 24 mandates, which is much less than in the 2021 election, where the surplus would have been 28, consisting of the 11 overhang mandates plus 17 mandatesthat the CSU would have received due to the second votes, separated from the district mandates.

[26] Here is in corner point 3 Proportionality on Federal prescribed. In the Simulations are based on Proportionality in the individual countries (here Bavaria).

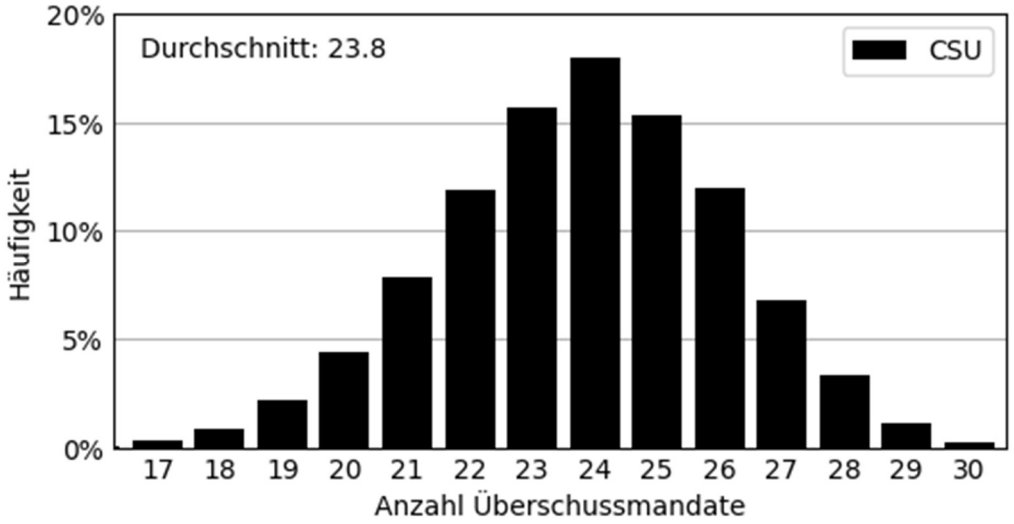

Figure 2: Surplus of the CSU party when applying the *trench systems*. Simulated results based on the 2021 Bundestag election in Bavaria. Surplus is, in a given distribution system, the number ofdistributed total mandates of a party – in this case CSU – beyond the contingent of seats.

The surplus is, of course, at the expense of other parties. Figure 3 shows how this surplus leads to "undershooting" for the other parties .

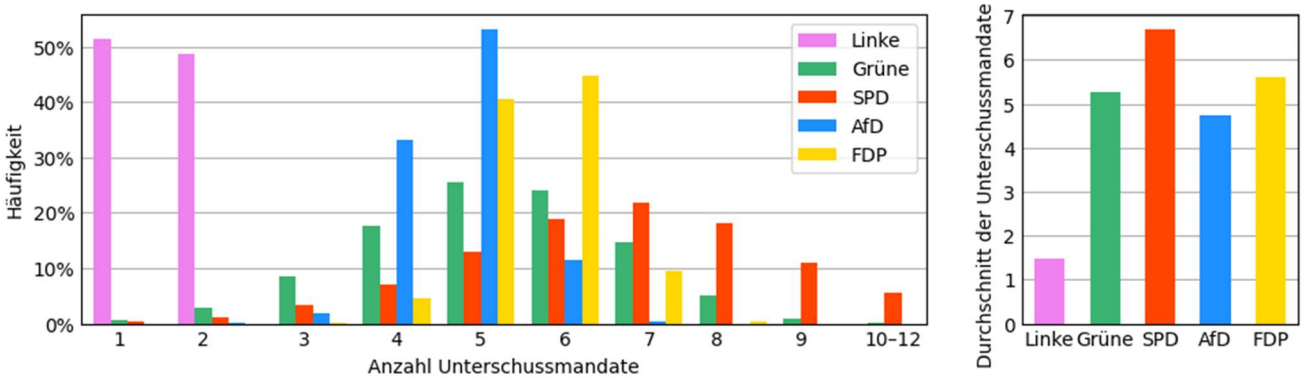

Figure 3: Subcontracting of the parties when applying the *trench system*. A party's understatement, in a given distribution system, is the number of seats that the party lacks to reach the contingent of seats. It can be seen that e.g. the mandates of the Bavarian Left Party in the durchaverage of all simulated election results are about 1.5 mandates below their second vote seat quota. In the left part of the figure you can see that in a good 50% of cases in which the party does not reach the seat quota target, only one mandate is missing. In the case of the SPD, it can be seen that the undershot is on average a good 6.5 (pillar right) and even in about 5% of the cases in which there is a deficit, the party lacks 10-12 seats.

In the *Scandinavian system*, the surplus of the CSU is much smaller than in the *trench system*, because the list mandates are distributed as *compensation mandates*, and do not benefit the CSU, see Figure 4. That is why the CSU's surplus remains at the 6.5 overhang mandates.

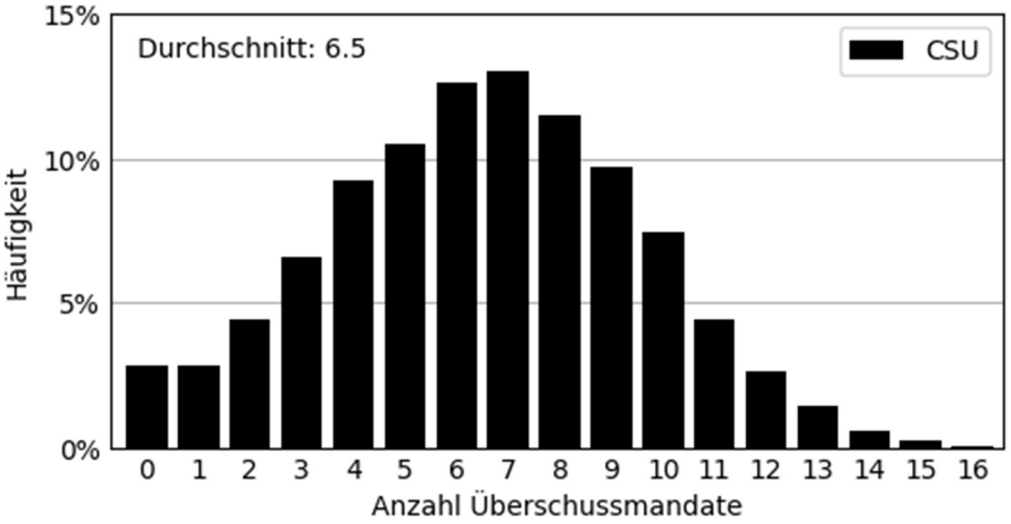

Figure 4: Csu party surplus when applying the *Scandinavian system*. Simulated results based on the 2021 Bundestag election in Bavaria. Surplus is, in a given distribution system, the number of distributed total mandates of a party – in this case CSU – beyond the seat quota. As stated in the main text, this figure is equal to Figure 1.

The other parties distribute all list mandates among themselves. Nevertheless, there is a deficit that is equal to the surplus of the CSU. overall. How this is distributed among the other parties can be seen in Figure 5.

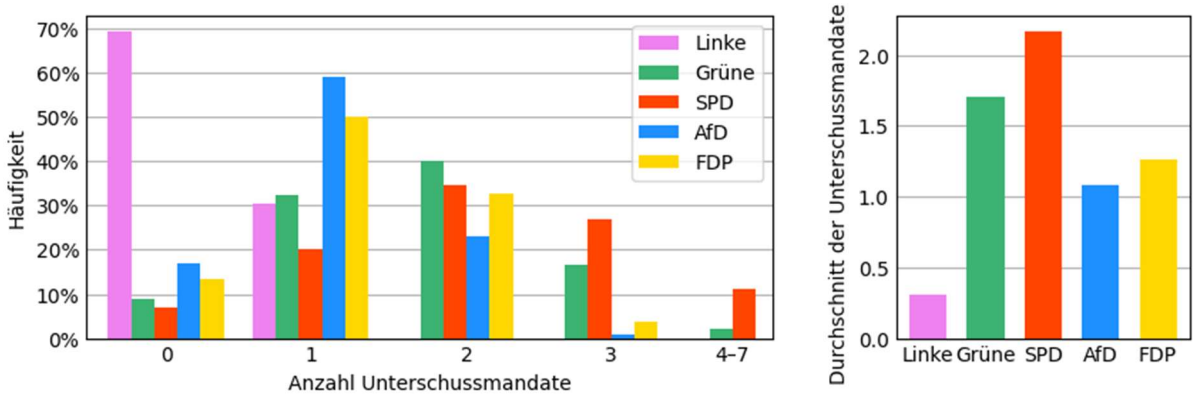

Figure 5**:** Shortfall of the parties in the application of the *Scandinavian system.* It canbe seen that the undershoot is now much smaller than when using the *trench system*. Overall, theaverage shortfall is 6.5 mandates. E.g. The SPD won just over 2.2 seats compared to the 6.5 seats in Figure 3. Nor do the same resultsoccur. In about 10% of cases where the party does not reach its seat quota, the missingmandates are never more than seven.

VII-3 Deviations from the majority election in the districts

The other side of the full compensation of disproportionality in the *traffic light systems* is that the district mandates do not always go to the strongest candidate. That's why it's interesting to see how often it happens that the strongest candidate is ignored. The question can already be read in Figure 1: On

average, in 6.5 of the 46 districts, a second strongest candidate must be chosen. [27] However, the districts areaffected by these deviations in different ways.

Figures 6 and 7 show in detail what the simulationssay about this phenomenon.

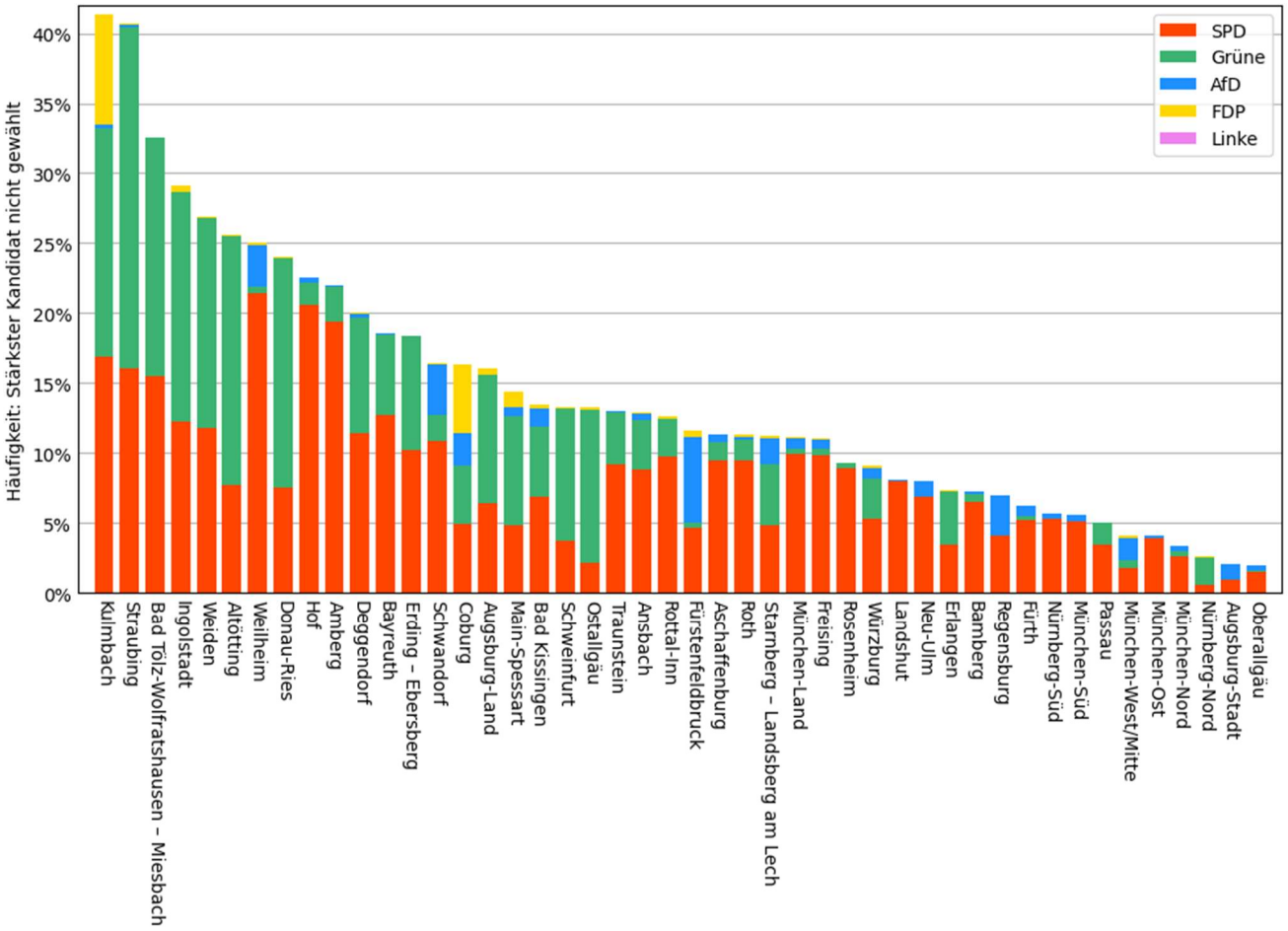

Figure 6: Deviations in the *corner variant* of the fact that the strongest candidate does not get the district mandate. The circles here are arranged according to the frequencyof this deviation. ==References== B. in about 42% of the simulated elections that the strongest candidate in the district of Kulmbach has to give way to the second strongest. This highest pillar also shows which parties get this mandate: SPD and Greens in about 16-17% of the elections and the FDP in about 9% of the cases. On the other hand, it happens in the s eltensten that thestrongestcandidate in the constituency of Oberallgäu does not get the mandate. The CSU has an average of 46.1% of the vote in the simulations, with the average of all votes of the CSU being 36.8% throughout Bavaria.

[27] Theoretically, the third couldStrengthenstoo Candidate (or a Weaker) can be considered. In the This never happens in simulations for Bavaria.

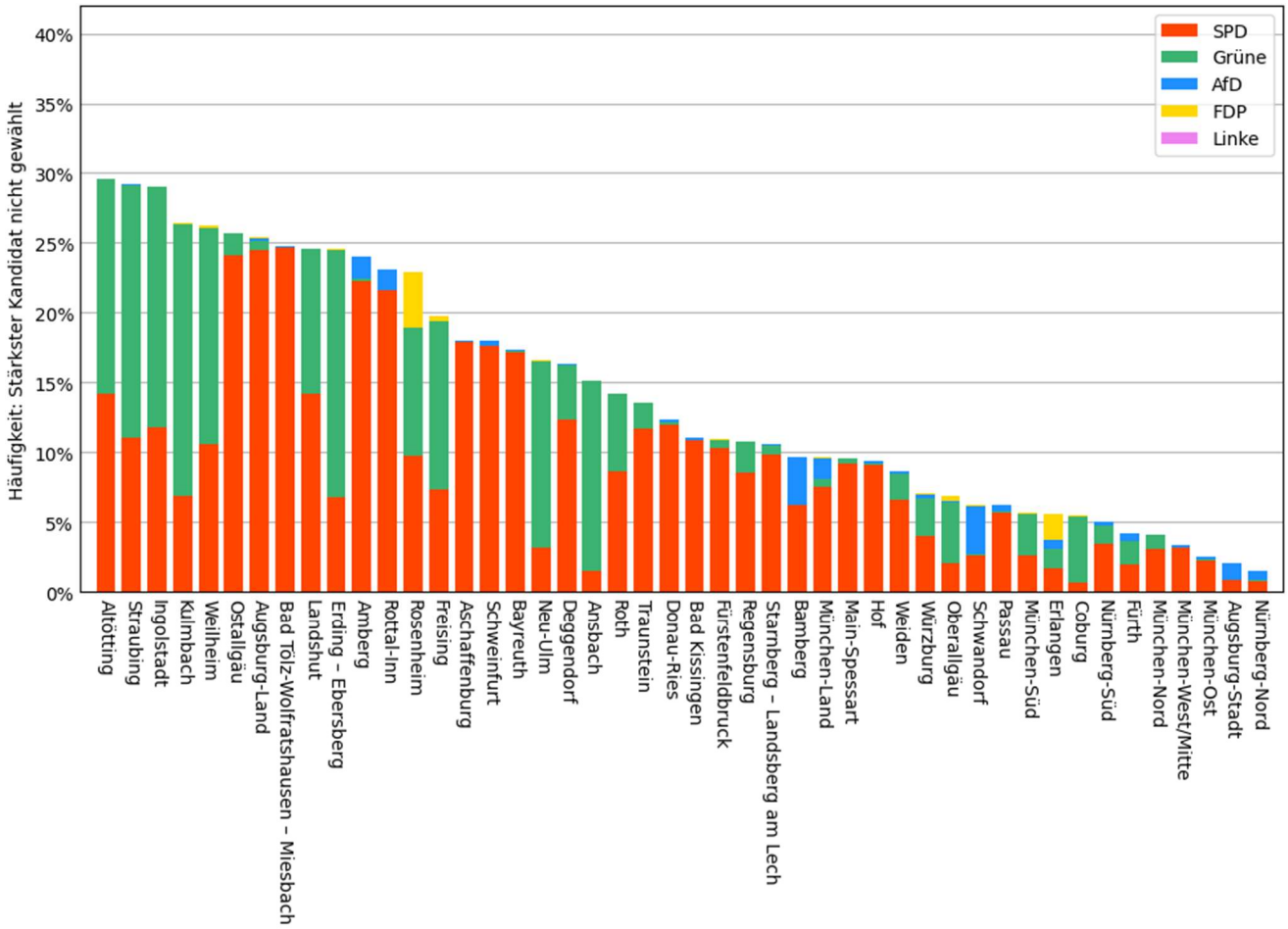

Figure 7: Deviations inm *Relativen advantage* of the fact that the strongest candidate does not get the kreismandat. The explanation of Figure 6 is also in place here. These two figures differconsiderably. Firstly, in just under 30% of cases, the maximum deviation is much smaller than that when using the *corner variant*. This means that the problem is less severe in the *advantagevariant*, because finally the sum of the deviations is the same for both, namely in the term 6.5 mandates, which corresponds to about 12.3% of all district mandates. Secondly, it is interesting becauses s the order of the circles is different. Under the *pretext* thront Altötting in the first place.

Another and important aspect is how big the difference is between the vote share of the strongest candidate and the proportion of the second strongestwho gets the mandate. This can be described as a 'vote deficit', although it is proportional to a percentage difference. Example: Ifthe strongest candidate in a districthas 35% of the vote, but the mandate still goes to the next with 27% of the vote, the end of the vote of the latter is calculated at $35\% - 27\% = 8\%$.

Figure 8 analyses how large this difference or deficit can be.

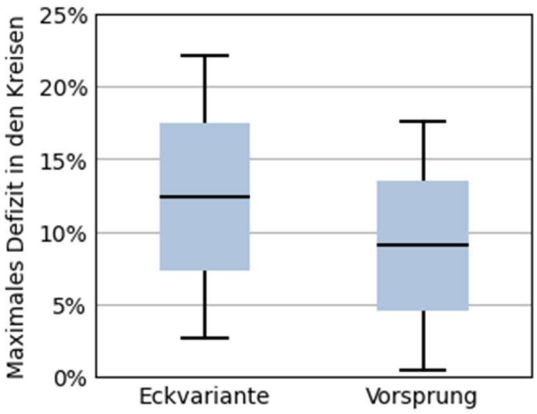

Figure 8: Large difference between the share of votes of the strongestcandidate and that of the voter in the two *traffic light variants*. By "maximum deficit" or "biggest difference" is meant the maximum across all circles in each of the simulations – and of that the average as always. Here itcan be seen that in the *corner variant* this maximum is on average 12.3%, but in the *preliminaryvariant* is much smaller, or 9.0%; see middle dashes. The blue boxes show the simulations that lie within a standarddeviation and the lower and upper dashes show the sizesof two standarddeviations. In addition, hardly any examplesare to be expected.

VII-4 Elected district candidates with the smallest share of the vote

This is still a quality feature, i.e. the smallest share of votes across all circles , which nevertheless leads to the mandate. It is probably desirable that this limit value is as high as possible. The average of all simulated election results of the smallest share of votes of an elected district candidate can be seen here for the four comparable systems:

	System			
	Graben	Skandinavisch	Eckvariante	Vorsprung
Kleinster Stimmenanteil eines Gewählten	27,3%	27,3%	21,1%	24,2%

In *the Trench* and *Scandinavian systems,* the candidate with the highest share of the vote is always elected. Nevertheless, on average, a candidate with the proportion down to 27.3% of the district votes must be reached. In the 2021 Bundestag election, the smallest share of the vote behind an elected district candidate in Bavaria was 29.2%.

In the *corner variant* and the *lead variant*, it cannot be avoided that such minimal share of the vote is smaller, because from time to time it is inevitable that the candidate with the largest share of the vote will not be elected. In the *corner variant,* on average, a candidate with 21.1% must be used. In the *advantage variant,* this minimum average is significantly higher, i.e. 24.2%. In Figure 9, the variations of these values are examined in more detail.

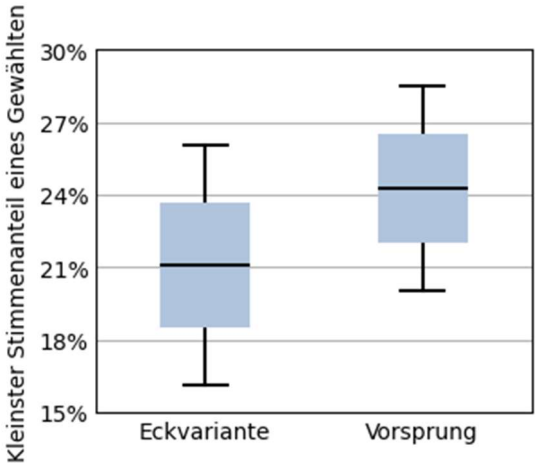

Figure 9: Smallest share of the votes of theselected circularsin the two *traffic light variants*. By "smallestm share of votes" is meant the Minimum over all circles in each of the simulations and the average of them. Otherwise, reference is made to the explanations in Figure 8.

VII-5 Differences between the systems

It is also of interest how big the difference is between the distribution of mandates in the different systems. This is shown in the following table:

	Einzelne Kreismandate				**Gesamtmandate der Parteien**			
	Graben	Skandinavisch	Eckvariante	Vorsprung	Graben	Skandinavisch	Eckvariante	Vorsprung
Graben			13,0	13,0		34,6	47,6	47,6
Skandinavisch			13,0	13,0	34,6		13,0	13,0
Eckvariante	13,0	13,0		8,3	47,6	13,0		
Vorsprung	13,0	13,0	8,3		47,6	13,0		

The left half of the table shows the overall difference in each circle. The first two systems differ from the two *traffic light variants* with an average of 13.0 mandates. This rhymes with what has already been said, namely that on average in 6.5 counties mandates do not go to the strongest candidate. This corresponds to a difference in two places in each of these circles; therefore twice 6.5 i.e. 13.0. The *traffic light variants* differ from each other with 8.3 mandates.

The overall difference between the parties is shown in the right half. There you can see that the *trench system* is very different from the other systems; as was said earlier.

VII-6 Entropy

The *entropy optimization method* is based on maximizing the logarithms of the chosen candidates. Therefore, this sum is a measure of how close the distribution in question is to the optimal distribution. The starting point is that no overhang is allowed. That's why this scale only makes sense for the traffic light systems. For the *corner variant,* the average value of this logarithm sum is equal to 503.309. For the *advantage variant*, it is 503.750. The difference may look small, as it is calculated. Nevertheless, the difference is significant in favor of the *advantage variant*.

As I said earlier, when writing this report, it was not possible to program the optimal method for the case that the seat quotas are based on second votes. Therefore, we cannot show the optimal value of the logarithm sum. If this had been successful, it would turn out how close the *lead variant is to* the optimal solution.

VII-7 Proportionality of overall mandates

Previously, this report focused mainly on the district mandates. The list mandates did not matter. In the *trench system*, they do. There, isolated from the district mandates, they are distributed proportionally among themselves , with the rule of Sainte-Laguë. In the *Scandinavian system*, the overall distribution – of district and list mandates – is proportional as long as there is no overhang, but this does not mean that the list mandates are distributed proportionally among themselves in isolation .

In the *traffic light variants*, all mandates are in and of themselves compensation mandates, i.e. they could be regarded as a bidimensional unit where the aim is to distribute them all proportionally among themselves . Then the list mandates are not a residual quantity, but a part of the overall distribution, and have just as "right" to the most proportionaldistribution as the district mandates.

The simulation of such a n biproportionaln overall distribution is in preparation.

VIII Summary

Four electoral systems that are applicable to elections to the Bundestag were examined. (1) The well-known *trench system*, (2) a system called *Scandinavian* here , (3) the system proposed in the *Commission's cornerstones*, here *corner variant*, and d (4) a variant of it, here marked as a *lead*.

In all four, a potential overhang is thwarted. The first two, however, do not achieve full proportional representation, which the other two achieve completely, but only with a deviation from the majority election in the districts. The systems were compared with different quality features shown in the following table:

Systems	Surplus	Max Deficit	Max. Defizithäufigkeit	Smallest share of votes
Dig	23,8	0%	0%	27%
Scandinavian	6,5	0%	0%	27%
Eckvariante	0,0	12%	41%	21%
Projection	0,0	9%	30%	24%

The figures show averages across all election results:

1. **Surplus** shows the number of seats that are above the seat quota of the parties (in Bavaria only with the CSU), whereby the seat quotas are based on the second votes.
2. **Deficit is the** largest difference in the share of votes of an elected person from the share of the strongest candidate in the same district across all constituencies.
3. **Max. Deficit frequency** refers to the circle where thestrongest candidate is most often elected.
4. **The smallest share** of votes across all constituencies is the smallest share of the vote of a candidate who is elected, be it the strongest or the Zstrongest in the district.

Surplus arises only in the *trench system* and in *Scandinavian*, but to a much lesser extent in the latter. Deficit is unavoidable in the *corner variant* and im *Vorsprung*, where it is milder. Smallest share of votes refers to all four systems. In the *trench system* and Scandinavian, the strongest candidate is always elected. Of these, the largest member has an vote share of only 27% in the passage. In the other systems, where the strongest candidate does not always get the mandate, the shares are of course smaller. Da, the *advantage variant* is also better than the *corner variant*.

Table 1. Overall distribution of mandates between Länder and parties on the basis of the resultsof the 2021 federal election

BP: Biproportionale Verteilung der Mandate nach Ländern und Parteien

	CDU	SPD	AfD	FDP	Linke	Grüne	CSU	SSW	Total
Schleswig-Holstein	5	6	2	3	1	4		1	22
Mecklenburg-Vorpommern	3	4	2	1	2	1			13
Hamburg	2	4	1	2	1	3			13
Niedersachsen	15	21	5	6	2	10			59
Bremen	1	2		1		1			5
Brandenburg	3	7	4	2	2	2			20
Sachsen-Anhalt	4	5	3	2	2	1			17
Berlin	4	6	2	3	3	6			24
Nordrhein-Westfalen	36	39	10	15	5	22			127
Sachsen	6	7	9	4	3	3			32
Hessen	11	13	4	6	2	7			43
Thüringen	3	4	4	2	2	1			16
Rheinland-Pfalz	8	10	3	4	1	4			30
Bayern		19	10	11	3	16	34		93
Baden-Württemberg	21	18	8	12	3	15			77
Saarland	2	3	1	1					7
Total	**124**	**168**	**68**	**75**	**32**	**96**	**34**	**1**	**598**

Erklärung des Farbcodes	**Weniger als in BP**	**Mehr als in BP**

PL: Verteilung der Mandate nach Parteien und innerhalb derer nach Ländern

	CDU	SPD	AfD	FDP	Linke	Grüne	CSU	SSW	Total
Schleswig-Holstein	6	7	2	3	1	5		1	25
Mecklenburg-Vorpommern	2	4	2	1	1	1			11
Hamburg	2	4	1	2	1	4			14
Niedersachsen	15	21	5	7	2	10			60
Bremen	1	1				1			3
Brandenburg	3	6	4	2	2	2			19
Sachsen-Anhalt	4	4	3	2	2	1			16
Berlin	4	6	2	2	3	6			23
Nordrhein-Westfalen	36	41	10	16	5	22			130
Sachsen	6	7	9	4	3	3			32
Hessen	11	13	4	6	2	7			43
Thüringen	3	4	4	2	2	1			16
Rheinland-Pfalz	8	10	3	4	1	4			30
Bayern		19	10	11	3	15	34		92
Baden-Württemberg	21	18	8	12	3	14			76
Saarland	2	3	1	1	1				8
Total	**124**	**168**	**68**	**75**	**32**	**96**	**34**	**1**	**598**

LP: Verteilung der Mandate nach Ländern und innerhalb derer nach Parteien

	CDU	SPD	AfD	FDP	Linke	Grüne	CSU	SSW	Total
Schleswig-Holstein	5	6	2	3	1	4		1	22
Mecklenburg-Vorpommern	2	4	3	1	2	1			13
Hamburg	2	4	1	2	1	3			13
Niedersachsen	15	21	5	6	2	10			59
Bremen	1	2		1		1			5
Brandenburg	3	7	4	2	2	2			20
Sachsen-Anhalt	4	5	3	2	2	1			17
Berlin	4	6	2	3	3	6			24
Nordrhein-Westfalen	35	40	10	15	5	22			127
Sachsen	6	7	9	4	3	3			32
Hessen	11	13	4	6	2	7			43
Thüringen	3	4	4	2	2	1			16
Rheinland-Pfalz	8	10	3	4	1	4			30
Bayern		20	10	11	3	15	34		93
Baden-Württemberg	21	18	8	13	3	14			77
Saarland	2	3	1	1					7
Total	**122**	**170**	**69**	**76**	**32**	**94**	**34**	**1**	**598**

Table 2. Vote numbers in Bavaria in the election to the Bundestag 2021

	Stimmenzahlen							Standardabweichungen in den Simulationen Beziehen sich auf einzelne Zahlen links. Dies gilt auch für sämtliche Totale.						
Erststimmen nach Wahlkreisen	**SPD**	**AfD**	**FDP**	**Linke**	**Grüne**	**CSU**	**Total**	**SPD**	**AfD**	**FDP**	**Linke**	**Grüne**	**CSU**	**Total**
Altötting	14 620	14 220	9 245	3 155	11 145	55 693	108 078	4 388	4 216	2 763	952	3 344	16 671	18 378
Erding – Ebersberg	24 205	11 448	13 226	3 355	24 840	70 656	147 730	7 236	3 476	3 942	996	7 370	20 756	23 665
Freising	25 950	18 042	14 687	3 898	24 058	69 689	156 324	7 824	5 519	4 442	1 176	7 319	20 529	24 233
Fürstenfeldbruck	36 831	13 056	16 029	3 216	25 363	72 721	167 216	11 090	3 897	4 859	969	7 706	21 991	26 395
Ingolstadt	25 954	17 806	10 877	4 648	18 182	83 663	161 130	7 614	5 402	3 274	1 405	5 412	25 380	27 738
München-Nord	38 172	7 591	19 477	6 216	42 319	44 854	158 629	11 522	2 282	5 889	1 856	12 898	13 202	22 818
München-Ost	38 243	8 066	18 104	4 907	42 367	61 159	172 846	11 517	2 445	5 275	1 494	12 573	18 031	25 712
München-Süd	33 924	7 641	16 437	6 236	47 256	46 059	157 553	10 149	2 307	4 841	1 870	14 103	13 711	22 532
München-West/Mitte	39 182	7 594	19 153	6 975	53 174	53 311	179 389	11 741	2 252	5 806	2 107	15 861	15 853	25 866
München-Land	30 237	9 816	18 180	3 685	40 475	77 523	179 916	9 155	2 954	5 537	1 116	12 311	23 534	28 393
Rosenheim	22 869	15 764	17 682	4 091	26 183	68 670	155 259	6 898	4 711	5 356	1 235	7 944	20 897	24 619
Bad Tölz-Wolfratshausen – Miesbach	15 428		11 636	2 643	20 829	55 501	106 037	4 702		3 503	779	6 153	16 619	18 642
Starnberg – Landsberg am Lech	23 985	10 715	16 585	3 701	35 809	68 617	159 412	7 139	3 253	5 029	1 116	10 854	20 495	25 185
Traunstein	27 644	12 671	12 268	3 537	17 219	59 555	132 894	8 288	3 804	3 705	1 087	5 199	17 967	21 164
Weilheim	19 682	9 686	8 553	2 718	16 300	57 179	114 118	5 837	2 862	2 581	821	4 882	17 095	19 206
Deggendorf	19 527	17 432	6 325	1 895	6 890	47 267	99 336	5 816	5 269	1 905	577	2 047	14 276	16 580
Landshut	22 468	19 184	22 774	3 955	21 811	70 685	160 877	6 646	5 801	6 854	1 160	6 514	21 256	24 933
Passau	28 341	16 215	10 513	2 681	12 098	41 530	111 378	8 543	4 843	3 143	801	3 644	12 659	16 836
Rottal-Inn	15 794	16 808	10 210	1 912	10 188	46 493	101 405	4 769	4 958	3 003	569	3 063	13 867	16 279
Straubing	16 312	16 794	6 895	1 936	8 613	58 487	109 037	4 878	5 048	2 100	587	2 601	17 551	19 132
Amberg	27 476	17 822	10 213	4 158	13 908	69 278	142 855	8 142	5 373	3 075	1 276	4 165	20 799	23 735
Regensburg	32 850	16 557	13 754	6 023	30 333	69 842	169 359	9 826	4 959	4 106	1 844	9 134	21 360	25 876
Schwandorf	39 615	23 142	7 341	3 043	7 934	60 924	141 999	11 756	6 861	2 191	911	2 383	18 203	23 154
Weiden	29 573	13 523	6 303	2 315	6 633	50 575	108 922	8 835	4 094	1 898	689	2 013	15 126	18 305
Bamberg	28 123	13 279	9 821	3 625	22 728	54 726	132 302	8 343	3 981	2 941	1 085	6 918	16 142	20 271
Bayreuth	24 840	10 777	9 182	2 577	14 064	54 465	115 905	7 400	3 234	2 776	780	4 276	16 545	19 166
Coburg	32 056	11 878	6 369	2 337	10 581	44 890	108 111	9 647	3 542	1 924	708	3 287	13 301	17 536
Hof	29 763	14 705	6 493	2 651	7 941	51 312	112 865	9 008	4 390	1 932	788	2 321	15 551	18 706
Kulmbach	22 103	14 416	6 480	2 448	10 165	65 163	120 775	6 538	4 399	1 923	743	3 034	19 405	21 325
Ansbach	33 819	18 418	13 016	3 798	21 343	73 312	163 706	10 167	5 547	3 870	1 152	6 482	22 235	26 242
Erlangen	32 036	10 669	10 382		29 923	54 223	137 233	9 663	3 182	3 122		9 065	16 471	21 502
Fürth	47 153	16 858	12 883	6 221	27 111	65 876	176 102	14 067	4 988	3 759	1 825	8 164	19 901	26 485
Nürnberg-Nord	31 616	8 485	10 882	7 726	32 541	41 027	132 277	9 567	2 541	3 300	2 350	9 757	12 329	19 129
Nürnberg-Süd	31 098	13 123	8 777	5 184	15 566	44 192	117 940	9 314	3 865	2 589	1 569	4 561	13 234	17 707
Roth	31 806	14 944	12 596	5 076	25 140	71 478	161 040	9 596	4 468	3 827	1 513	7 501	21 081	24 816
Aschaffenburg	24 893	13 954	11 683	3 536	19 588	59 269	132 923	7 422	4 214	3 528	1 046	5 930	18 294	21 325
Bad Kissingen	32 844	17 130	12 206	4 475	16 467	67 458	150 580	10 090	5 201	3 703	1 354	4 883	20 489	24 470
Main-Spessart	33 700	12 576	9 514	4 070	15 813	60 489	136 162	10 084	3 729	2 866	1 192	4 769	18 200	21 935
Schweinfurt	29 037	15 468	12 408	6 552	14 747	63 697	141 909	8 907	4 675	3 719	1 952	4 342	18 634	22 112
Würzburg	33 125		19 414	7 522	36 295	67 651	164 007	9 848		5 851	2 297	10 711	20 108	25 755
Augsburg-Stadt	27 453	13 431	12 880	7 168	31 347	42 780	135 059	8 364	4 074	3 873	2 135	9 295	12 726	18 664
Augsburg-Land	29 435	19 660	16 032	3 573	24 806	82 423	175 929	8 799	5 942	4 751	1 080	7 332	24 439	27 864
Donau-Ries	29 872	15 723	9 915	3 048	11 076	64 045	133 679	8 859	4 749	2 936	911	3 351	19 102	21 836
Neu-Ulm	29 960	22 517	14 542	3 466	20 621	69 676	160 782	9 044	6 722	4 341	1 019	6 326	20 672	24 793
Oberallgäu	28 401	14 473	23 604	4 911	27 817	53 566	152 772	8 585	4 381	7 033	1 463	8 167	16 249	22 040
Ostallgäu	24 288	20 021	14 258	6 667	24 128	76 399	165 761	7 375	6 031	4 220	1 991	7 156	22 636	25 793
Die Kreise zusammen	1 316 303	634 098	579 804	187 530	1 023 735	2 788 048	6 529 518	58 588	30 013	27 206	9 040	50 027	123 328	151 388
Zweitstimmen	1 361 242	679 915	798 591	210 838	1 067 830	2 402 827	6 521 243	136 123	67 427	79 143	20 673	107 069	241 151	310 632

Table 3. Order of district mandates based on the number of votes in Bavaria in the election to the Bundestag in 2021. The first 34 seats will be awarded to the CSU in both methods, but not in the same constituencies

	Eckvariante				Relativer Vorsprung			
	Wahlkreise	Partei	Stimm-Anteile		Wahlkreise	Größte Partei	Zweitgrößte Partei	Verhältnis
1	Kulmbach	CSU	54,0%	1	Altötting	CSU	SPD	3,81
2	Straubing	CSU	53,6%	2	Straubing	CSU	AfD	3,48
3	Bad Tölz-Wolfratshausen – Miesbach	CSU	52,3%	3	Ingolstadt	CSU	SPD	3,22
4	Ingolstadt	CSU	51,9%	4	Ostallgäu	CSU	SPD	3,15
5	Altötting	CSU	51,5%	5	Landshut	CSU	FDP	3,10
6	Weilheim	CSU	50,1%	6	Kulmbach	CSU	SPD	2,95
7	Amberg	CSU	48,5%	7	Weilheim	CSU	SPD	2,91
8	Donau-Ries	CSU	47,9%	8	Erding – Ebersberg	CSU	Grüne	2,84
9	Erding – Ebersberg	CSU	47,8%	9	Augsburg-Land	CSU	SPD	2,80
10	Deggendorf	CSU	47,6%	10	Rottal-Inn	CSU	AfD	2,77
11	Bayreuth	CSU	47,0%	11	Freising	CSU	SPD	2,69
12	Augsburg-Land	CSU	46,9%	12	Bad Tölz-Wolfratshausen – Miesbach	CSU	Grüne	2,67
13	Weiden	CSU	46,4%	13	Rosenheim	CSU	Grüne	2,62
14	Ostallgäu	CSU	46,1%	14	Amberg	CSU	SPD	2,52
15	Rottal-Inn	CSU	45,8%	15	Deggendorf	CSU	SPD	2,42
16	Hof	CSU	45,5%	16	Aschaffenburg	CSU	SPD	2,38
17	Schweinfurt	CSU	44,9%	17	Neu-Ulm	CSU	SPD	2,33
18	Traunstein	CSU	44,8%	18	Roth	CSU	SPD	2,25
19	Bad Kissingen	CSU	44,8%	19	Schweinfurt	CSU	SPD	2,19
20	Ansbach	CSU	44,8%	20	Bayreuth	CSU	SPD	2,19
21	Aschaffenburg	CSU	44,6%	21	Ansbach	CSU	SPD	2,17
22	Freising	CSU	44,6%	22	Traunstein	CSU	SPD	2,15
23	Main-Spessart	CSU	44,4%	23	Donau-Ries	CSU	SPD	2,14
24	Roth	CSU	44,4%	24	Regensburg	CSU	SPD	2,13
25	Rosenheim	CSU	44,2%	25	Bad Kissingen	CSU	SPD	2,05
26	Landshut	CSU	43,9%	26	Fürstenfeldbruck	CSU	SPD	1,97
27	Fürstenfeldbruck	CSU	43,5%	27	Bamberg	CSU	SPD	1,95
28	Neu-Ulm	CSU	43,3%	28	Starnberg – Landsberg am Lech	CSU	Grüne	1,92
29	München-Land	CSU	43,1%	29	München-Land	CSU	Grüne	1,92
30	Starnberg – Landsberg am Lech	CSU	43,0%	30	Oberallgäu	CSU	SPD	1,89
31	Schwandorf	CSU	42,9%	31	Würzburg	CSU	Grüne	1,86
32	Coburg	CSU	41,5%	32	Main-Spessart	CSU	SPD	1,80
33	Bamberg	CSU	41,4%	33	Hof	CSU	SPD	1,72
34	Würzburg	CSU	41,2%	34	Weiden	CSU	SPD	1,71
35	München-Süd	Grüne	30,0%	35	Coburg	SPD	AfD	2,70
36	München-West/Mitte	Grüne	29,6%	36	Nürnberg-Süd	SPD	Grüne	2,00
37	Fürth	SPD	26,8%	37	Passau	SPD	AfD	1,75
38	München-Nord	Grüne	26,7%	38	Fürth	SPD	Grüne	1,74
39	Nürnberg-Süd	SPD	26,4%	39	Schwandorf	SPD	AfD	1,71
40	Passau	SPD	25,4%	40	München-Süd	Grüne	SPD	1,39
41	Nürnberg-Nord	Grüne	24,6%	41	München-West/Mitte	Grüne	SPD	1,36
42	München-Ost	Grüne	24,5%	42	Augsburg-Stadt	Grüne	SPD	1,14
43	Erlangen	SPD	23,3%	43	München-Nord	Grüne	SPD	1,11
44	Augsburg-Stadt	Grüne	23,2%	44	München-Ost	Grüne	SPD	1,11
45	Regensburg	SPD	19,4%	45	Erlangen	SPD	Grüne	1,07
46	Oberallgäu	SPD	18,6%	46	Nürnberg-Nord	Grüne	SPD	1,03

Table 4. "Corrective" distribution of district mandates in the relative *advantage* method

Gesamtzahlen von Sitzen der Parteien

Partei	Kreis-mandate	Sitz-koningent	Differenz: Überhang, wenn positiv
SPD	0	20	-20
AfD	0	10	-10
FDP	0	11	-11
Linke	0	3	-3
Grüne	1	15	-14
CSU	45	34	11

Abschaffung des Überhangs durch Wechseln von Mandaten

Reihen-folge	Kreis in dem gewechselt wird	Zu	Von	Verhältnis der Stimmen Zu/Von
1	München-West/Mitte	Grüne	CSU	0,997
2	München-Nord	Grüne	CSU	0,943
3	Nürnberg-Nord	Grüne	CSU	0,793
4	Augsburg-Stadt	Grüne	CSU	0,733
5	Fürth	SPD	CSU	0,716
6	Coburg	SPD	CSU	0,714
7	Nürnberg-Süd	SPD	CSU	0,704
8	München-Ost	Grüne	CSU	0,693
9	Passau	SPD	CSU	0,683
10	Schwandorf	SPD	CSU	0,650
11	Erlangen	SPD	CSU	0,591

Table5a. Explanatory example

Daten	**Erststimmen**	**I**	**II**	**III**	**Total**	**Größte Stimmenzahlen**	**Erklärungen**
Kreise	Ost	70	120	200	390	200	Drei Wahlkreise: Ost, Mitte und West
	Mitte	50	125	200	375	200	Drei Parteien: I, II und III
	West	200	190	50	440	200	Vier Listenmandate
	Total	320	435	450	1205		
	Erststimmen-anteile	**I**	**II**	**III**	**Total**	**Größte Anteile**	
	Ost	18%	31%	51%	100%	51%	
	Mitte	13%	33%	53%	100%	53%	
	West	45%	43%	11%	100%	45%	
	Total	27%	36%	37%	100%		
	Zweitstimmen	**I**	**II**	**III**	**Total**		
Landeslisten		550	445	220	1215		
	Zweitstimmen-anteile	**I**	**II**	**III**	**Total**		
		45%	37%	18%	100%		
		I	**II**	**III**	**Mandate**		
	Sitzkontingente	3	3	1	7		Verteilung aller sieben Sitze (3+4) an die Parteien, allein auf den Zweitstimmen basiert.
	Potentieller Überhang			1	1		Wenn die Parteien (hier Partei III) mehr Mandate bekommen als in den Sitzkontingenten vorgesehen droht ein Überhang.

Grabensystem

		I	**II**	**III**	**Mandate**	
Kreise	Ost			1	1	Mehrheitswahl aufgrund der Stimmenzahlen.
	Mitte			1	1	
	West	1			1	
	Total	1		2	3	
Listenmandate gem. Zweitstimmen		2	1	1	4	Verteilung der Listenmandate, basiert auf den Zweitstimmen, unabhängig von den Kreismandaten.
Gesamtmandate		3	1	3	7	Summe der direkten Kreismandate und Listenmandate.
Sitzkontingent			-2	2		Abweichung von rein proportionaler Gesamtverteilung.

Skandinavisches System

		I	**II**	**III**	**Mandate**	
Kreise	Ost			1	1	Mehrheitswahl aufgrund der Stimmenzahlen.
	Mitte			1	1	
	West	1			1	
	Total	1		2	3	
Listenmandate als Differenz		2	2		3	Differenzen von gerechneten Gesamtmandaten (s. nächste Zeile) und den Kreismandaten.
Gesamtmandate		3	2	2	7	Die Gesamtzahl der Sitze (7) minus die Kreismandate der Überhang-partei (III) werden auf die anderen Parteien verteilt (5 Mandate).
Abweichung vom Sitzkontingent			-1	1		Abweichung von rein proportionaler Gesamtverteilung.

Ampelsystem; Eckvariante

	Höchste Stimmenanteile in jedem Kreis					**Höchste Anteile**	
		I	**II**	**III**			
Kreise	Ost			51%		51%	Zuerst wird herausgefunden welche Kreiskandidaten die höchsten Stimmenanteile haben.
	Mitte			53%		53%	
	West	45%				45%	
	Verteilung nach den höchsten Anteilen in jedem Kreis						
		I	**II**	**III**	**Mandate**		
	Ost						Diese Verteilung ist begrenzt durch die Sitzkontingente.
	Mitte			1	1		Darum geht der Kreis "Ost" zuerst leer aus.
	West	1			1		
	Zweithöchste Stimmenanteile in jedem Kreis (hier nur einem)					**Zweithöchster Anteil**	
		I	**II**	**III**			
	Ost		31%			31%	Im Kreis "Ost" wird zu dem Kandidaten mit dem zweitgrößten Stimmenanteil gegriffen.
	Mitte						
	West						
	Endgültige Mandatverteilung						
		I	**II**	**III**	**Mandate**		
	Ost		1		1		Endgültige Verteilung der Kreismandate.
	Mitte			1	1		
	West	1			1		
	Total	1	1	1	3		
Listenmandate als Differenz		2	2		4		Differenzen von gerechneten Gesamtmandaten und den Kreismandaten.
Gesamtmandate, gleich Sitzkontingenten		3	3	1	7		Stimmt immer mit den Sitzkontingenten überein.
Entropie der Kreismandate	15,384						Desto grösser diese Zahl ist, desto besser!

Table 5b. Explanatory example, continued

Ampelsystem; Relativer Vorsprung

		Größte Stimmenzahlen in jedem Kreis				Größte	
		I	II	III		Stimmenzahlen	
Kreise	Ost			200		200	Zuerst wird herausgefunden, welche Kandidaten die größte Anzahl
	Mitte			200		200	von Stimmen in jedem Kreis haben.
	West	200				200	
	Zweitgrößte Stimmenzahl in jedem Kreis					**Zweitgrößte**	
		I	**II**	**III**		**Stimmenzahlen**	
	Ost		120			120	Auch die Kandidaten der zweitgrößten Anzahl von Stimmen
	Mitte		125			125	müssen hervorgehoben werden.
	West		190			190	
	Drittgrößte Anzahl von Stimmen in jedem Kreis					**Drittgrößte**	
		I	**II**	**III**		**Stimmenzahlen**	
	Ost	70				70	Und sogar die Kandidaten mit dem dritthöchsten Stimmenanteil
	Mitte	50				50	können eine Rolle spielen.
	West			50		50	
	Verhältnisse größter zu zweitgrößten					**Die größten**	
		I	**II**	**III**		**dieser**	
	Ost			1,667		1,667	Verhältnis zwischen der größten und der zweitgrößten
	Mitte			1,600		1,600	Stimmenzahl wird in jedem Kreis ermittelt.
	West	1,053				1,053	
	Verteilung nach den größten Verhältnissen						
		I	**II**	**III**	**Mandate**		
	Ost			1	1		Diese Verteilung ist begrenzt durch die Sitzkontingente.
	Mitte						Darum geht der Kreis "Mitte" zuerst leer aus.
	West	1			1		
	Zweitgrößte Verhältnisse in jedem Kreis (hier nur einem)					**Zweitgrößtes Verhältnis**	
		I	**II**	**III**			
	Ost						
	Mitte		2,500			2,500	Im Kreis "Mitte" wird zu dem Kandidaten mit dem höchsten Stimmenanteil gegriffen.
	West						
	Endgültige Mandatverteilung						
		I	**II**	**III**	**Mandate**		
	Ost			1	1		Endgültige Verteilung der Kreismandate.
	Mitte		1		1		
	West	1			1		
	Total	1	1	1	3		
Listenmandate als Differenz		2	2		4		Differenzen von gerechneten Gesamtmandaten und den Kreismandaten.
Gesamtmandate, gleich Sitzkontingenten		3	3	1	7		Stimmt immer mit den Sitzkontingenten überein.
Entropie der Kreismandate	**15,425**						Desto grösser diese Zahl ist, desto besser!

Ampelsystem; Optimale Lösung

		Entropie: Logarithmen der Stimmen				
		I	**II**	**III**		
Kreise	Ost	4,25	4,79	5,30		Die optimale Methode beruht darauf (in diesem einfachen
	Mitte	3,91	4,83	5,30		Beispiel), diejenige Verteilung zu ermitteln, durch welche die
	West	5,30	5,25	3,91		Summe der Logarithmen der Stimmen der gewählten Kandidaten
	Endgültige Mandatverteilung					
		I	**II**	**III**	**Mandate**	
	Ost			1	1	Ergebnis der Optimierung der Entropie der möglichen
	Mitte		1		1	Kreismandatverteilungen
	West	1			1	
	Total	1	1	1	3	
Listenmandate als Differenz		2	2		4	Differenzen von gerechneten Gesamtmandaten und den Kreismandaten.
Gesamtmandate		3	3	1	7	
Entropie der Kreismandate	**15,425**					Die Entropie ist die Summe der Logarithmen der Stimmen der gewählten Kandidaten. Die Optimale Lösung liefert immer die

Überhangsystem

	I	II	III	Mandate	
Kreismandate	1		2	3	So wie im Grabensystem und dem Skandinavischen.
Listenmandate als Differenz	2	3		3	Differenzen von gerechneten Gesamtmandaten und den Kreismandaten.
Gesamtmandate	3	3	2	8	Maximum von Direktmandaten und den Sitzkontingenten.
Überhang			1	1	Abweichung von den Sitzkontingenten, d.h. Überhang.

Ausgleichssystem

	I	II	III	Mandate	
Kreismandate	1		2	3	Alles wie im Überhangsystem.
Listenmandate als Differenz	3	3		3	
Mandate mit Überhang	3	3	2	8	
Ausgleichsmandate	1			1	Notwendig, um volle Proportionalität zwischen den Parteien zu erreichen.
Gesamtmandate	4	3	2	9	Endgültige Gesamtverteilung.